Branes, Superpotentials and Superconformal Fixed Points

Ofer Aharony[1] and Amihay Hanany[2]

[1]Department of Physics and Astronomy,

Rutgers University

Piscataway, NJ 08855-0849, USA

`oferah@physics.rutgers.edu`

[2]School of Natural Sciences

Institute for Advanced Study

Princeton, NJ 08540, USA

`hanany@ias.edu`

We analyze various brane configurations corresponding to field theories in three, four and five dimensions. We find brane configurations which correspond to three dimensional $N = 2$ and four dimensional $N = 1$ supersymmetric QCD theories with quartic superpotentials, in which what appear to be "hidden parameters" play an important role. We discuss the construction of five dimensional $N = 1$ supersymmetric gauge theories and superconformal fixed points using branes, which leads to new five dimensional $N = 1$ superconformal field theories. The same five dimensional theories are also used, in a surprising way, to describe new superconformal fixed points of three dimensional $N = 2$ supersymmetric theories, which have both "electric" and "magnetic" Coulomb branches.

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1. Introduction

In the past year many interesting results, both in field theory and in string theory, were discovered by studying the worldvolume dynamics of branes in superstring theories. A particular construction was used in [1] to study the dynamics of $N = 4$ supersymmetric gauge theories in $2 + 1$ dimensions. Variants of this construction have been applied in [2] to $N = 1$ theories in $3 + 1$ dimensions and in [3] to solving a large class of $N = 2$ theories in $3 + 1$ dimensions.

In this paper we analyze several new generalizations of the brane construction of [1], and discuss various problems which arise in the translation from the brane construction to the field theory. We begin in section 2 by analyzing 3D $N = 2$ theories which are related by rotations of the branes to the theories discussed in [1]. We find that the "hidden parameters" discussed in [1], corresponding to the x_6 positions of the D-branes, are no longer hidden, but affect the superpotential in some configurations. The precise form of this effect is not clear, but we provide consistent superpotentials for describing the low-energy theory of general brane configurations. We discuss also the generalization of these results to 4D $N = 1$ theories, and their consistency with the brane constructions of mirror symmetry [4,1], Seiberg duality [5,2] and chiral symmetry [6].

In section 3, which is independent of section 2, we discuss the generalization of the construction of [1] to five dimensional field theories with $N = 1$ supersymmetry. We find many interesting effects, related to the recent analysis of 5D $N = 1$ field theories in [7,8,9,10], and provide (implicit) constructions for new 5D $N = 1$ superconformal field theories.

In section 4 we use the brane constructions of section 3 with additional D3-branes to construct new 3D $N = 2$ superconformal field theories, which have both "electric" and "magnetic" Coulomb branches emanating from a single point in their moduli space. Every 5D superconformal theory we construct in section 3 gives rise to a 3D $N = 2$ superconformal theory, opening a way to a systematic study of a large class of new three dimensional theories.

2. Branes and Superpotentials in 3D $N = 2$ Supersymmetric Gauge Theories

2.1. The brane construction

In [1] a configuration of 3-branes and 5-branes was used to construct $N = 4$ supersymmetric gauge theories in three dimensions. In this section we will generalize this brane

configuration to a construction of $N = 2$ gauge theories in three dimensions. First, we recall the configuration of [1]. The configuration is in the type IIB string theory with time coordinate x_0 and space coordinates $x_1, \ldots, x_9$. We denote by Q_L and Q_R the supercharges generated by left- and right- moving world-sheet degrees of freedom. The type IIB theory is chiral, and the supercharges obey $\bar{\Gamma}Q_L = Q_L$ and $\bar{\Gamma}Q_R = Q_R$ with $\bar{\Gamma} = \Gamma_0 \cdots \Gamma_9$. The brane configuration of [1] includes NS 5-branes which span a worldvolume in the 012345 directions, D5-branes which span a worldvolume in the 012789 directions and 3-branes which span a worldvolume in the 0126 directions. This configuration preserves 1/4 of the supersymmetry. The 3-branes are finite in one direction and thus their worldvolume theory leads to $N = 4$ supersymmetry in three dimensions. Breaking the supersymmetry by a further half may be achieved by introducing a rotation angle [11] to at least one of the branes. We can choose the rotation to be in the 45 - 89 space, i.e. rotate by an angle θ in the $x_4 - x_8$ and $x_5 - x_9$ planes. In fact, we can rotate each of the 5-branes by different rotation angles without breaking further the supersymmetry (below 3D $N = 2$)[1]. We will discuss general situations of this sort. In a particular case where we choose the angles of the NS 5-branes to be 0 and 90 degrees while the angles of the D5-branes are zero (relative to the configuration of [1] described above) the configuration is T-dual (in the x_3 direction) to the configuration described in [2]. This configuration was also studied in [13].

In the special cases where the NS 5-branes have zero angle, i.e. with worldvolume coordinates spanning the 012345 directions, we will call the 5-brane a NS brane, while for 90 degrees, i.e. worldvolume coordinates spanning the 012389 directions, we will call the 5-brane a NS$'$ brane. Similarly, when the D5-branes have zero angle with worldvolume coordinates 012789 we will call them D 5-branes, and for 90 degrees with worldvolume coordinates 012457 we will call them D$'$ 5-branes.

The unbroken supersymmetry generators are linear combinations $\epsilon_L Q_L + \epsilon_R Q_R$ which satisfy

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\epsilon_L$$
$$\epsilon_R = -\Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\epsilon_R$$

(2.1)

due to the presence of a NS brane,

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_8\Gamma_9\epsilon_L$$
$$\epsilon_R = -\Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_8\Gamma_9\epsilon_R$$

(2.2)

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[1] This was also discussed in [12].

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due to the presence of a NS′ brane,

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_7\Gamma_8\Gamma_9\epsilon_R \qquad (2.3)$$

due to the presence of a D 5-brane,

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_4\Gamma_5\Gamma_7\epsilon_R \qquad (2.4)$$

due to the presence of a D′ 5-brane, and

$$\epsilon_L = \Gamma_0\Gamma_1\Gamma_2\Gamma_6\epsilon_R \qquad (2.5)$$

due to the presence of the D3-branes. Additional branes can also be added without breaking the supersymmetry any further, but we will not discuss them here.
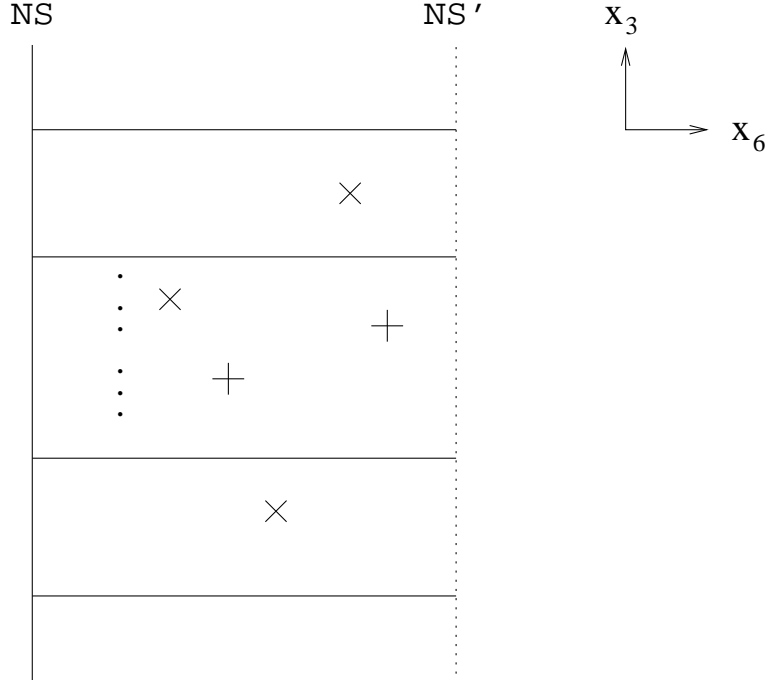

Figure 1: Three dimensional $N = 2$ supersymmetric gauge theories with gauge group $U(N_c)$ and N_f quarks. There are N_c D3-branes (horizontal lines), which are stretched in between two NS 5-branes (vertical lines). The figure is depicted in the 36 plane as indicated by the arrows in the upper right of the figure. The left 5-brane stretches along the 012345 directions and is denoted NS and the right 5-brane stretches along the 012389 directions and is denoted NS′. The "X"s denote D5-branes, and the "+"s denote D′ 5-branes, both of which give rise to quarks.

The presence of all these branes breaks the Lorentz group $SO(1,9)$ to $SO(1,2) \times SO(2)_{45} \times SO(2)_{89}$, where $SO(1,2)$ acts on x_0, x_1 and x_2, $SO(2)_{45}$ acts as rotations in the x_4, x_5 plane and $SO(2)_{89}$ acts as rotations in the x_8, x_9 plane. The $SO(2)$ symmetries are broken when some of the branes are at angles different from 0 or 90 degrees.

As in [1] we introduce the mirror symmetry operation. The S transformation in the $SL(2, \mathbb{Z})$ U-duality group of the type IIB string theory, given by the matrix

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\tag{2.6}$$

together with the rotation which maps x_i to x_{i+4} and x_{i+4} to $-x_i$ ($i = 3, 4, 5$), define an operation which we call mirror symmetry.

2.2. The field content of the 3D theory

As in [1] we will take the D3-branes to be finite in one direction. Their low energy theory (below the scale determined by the distance between the NS 5-branes) is then described by a three dimensional field theory. One might expect this theory to be the dimensional reduction of the 10D SYM theory, which gives $N = 8$ SYM in three dimensions. However, as in [1], boundary conditions on some of the fields where the D3-brane ends on the 5-branes remove these fields from the low energy theory.

The boundary condition for a D3-brane ending on a NS(012345) 5-brane projects out the motions of the D3-brane in the x_7, x_8 and x_9 directions, as well as the A_6 component of the D3-brane gauge field. The 789 components are set to the values of the position of the NS brane in these directions, and A_6 is also frozen (but it is not clear at which value). The boundary conditions coming from a NS(012389) 5-brane project out the x_4, x_5 and x_7 scalars in the field theory, as well as the A_6 component of the gauge field. The 457 components are set to the values of the position of the NS brane in these directions. In order to generalize these results to NS 5-branes at arbitrary angles, we introduce complex coordinates $z_1 = x_4 + ix_5$ and $z_2 = x_8 + ix_9$. The 3-brane worldvolume theory will include scalar fields ϕ_1 and ϕ_2, respectively, which denote the position of the 3-brane in these directions. These are the scalar components of chiral multiplets in the 3D $N = 2$ theory. Two more scalar parameters in the D3-brane worldvolume theory are the x_7 coordinate and the A_6 component of the gauge field. To understand what kind of multiplet these scalars form we recall that mirror symmetry exchanges the scalars ϕ_1 and ϕ_2. Mirror symmetry also acts as electric-magnetic duality on the worldvolume of the 3-brane. Thus,

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it exchanges the x_7 coordinate of the 3-brane and the x_6 component of the gauge field with the x_3 coordinate of the 3-brane and the scalar dual to the 3D gauge field, respectively. The scalars x_7 and A_6 are dual to a vector multiplet which includes A_μ and x_3. To some extent the scalar $x_7 + iA_6$ can be treated as the lowest component of a chiral multiplet ϕ_3. There is a $U(1)$ symmetry which acts by shifting the A_6 component of the gauge field (which is periodic in the quantum theory).

The boundary conditions for a D3-brane ending on a NS 5-brane set ϕ_3 and a linear combination of ϕ_1 and ϕ_2 (which depends on the angle of the NS 5-brane) to zero at the boundary. The boundary conditions for a D3-brane ending on a D5-brane likewise set to zero the vector multiplet and a linear combination of ϕ_1 and ϕ_2 depending on the angle of the D5-brane. From the three dimensional point of view, each of these fields will give rise to a tower of massive states corresponding to the momenta in the x_6 direction, the lowest of which will have a mass of order $1/|\Delta x_6|$ where Δx_6 is the span of the D3-brane in the x_6 direction. The relation of Δx_6 to the gauge coupling [1] sets this mass to be of the order of g^2, where g is the gauge coupling. In [1] (and in other cases discussed in the literature), the presence of these massive states was not important for the low-energy field theory. However, as we will see in §2.4, in some cases these states will give rise to important interactions in the low-energy field theory, and they cannot be ignored.

Now, we can consider which fields remain massless in the D3-brane worldvolume theory, beginning with the vector multiplet and ϕ_3. As in [1] there are three cases to consider :

1) When a 3-brane has both ends on NS 5-branes, the $N = 2$ vector multiplet survives while the chiral multiplet ϕ_3 is frozen (i.e. massive). In this case the x_7 coordinates of the 5-branes must coincide to preserve supersymmetry. The x_6 distance between the two NS 5-branes is identified with $1/g^2$ (up to the string coupling), and the x_7 distance between them is identified with the Fayet-Iliopoulos term for the gauge field.

2) When a 3-brane has both ends on D5-branes, the $N = 2$ chiral multiplet ϕ_3 survives while the vector multiplet is frozen. In this case the x_3 coordinates of the D5-branes must coincide to preserve supersymmetry.

3) When a 3-brane ends on a D5-brane at one side and on a NS 5-brane at the other side both ϕ_3 and the vector multiplet are frozen.

We need also to see what are the remaining massless fields which are associated with the positions along the 45 and 89 directions. For this the analysis does not distinguish between the various cases described above. Given a 3-brane with both ends on 5-branes

whose 45-89 position is given by an equation $az_1 + bz_2 = \alpha$ for the first 5-brane, and an equation $cz_1 + dz_2 = \beta$ for the other 5-brane, there are several cases to consider :

i) If $ad - bc \neq 0$ there is a unique solution to the two equations, and both scalar fields are frozen to the values of the solution. There are no additional moduli beyond the ones described above.

ii) If $ad - bc = 0$ with a one parameter solution to the two equations, there is one massless chiral multiplet corresponding to the motion of the D3-brane in the direction perpendicular to $az_1 + bz_2$. This chiral multiplet combines with the other multiplets considered in cases 1) to 3) above to form :

1) an $N = 4$ vector multiplet;

2) an $N = 4$ hypermultiplet;

3) an $N = 2$ chiral multiplet.

iii) If $ad - bc = 0$ with no solution to the two equations, there is no supersymmetric configuration of a D3-brane between the two 5-branes. Such a situation corresponds, as in [1], to turning on Fayet-Iliopoulos parameters for the $N = 4$ gauge theory (which look like a superpotential proportional to $b\phi_1 - a\phi_2$ in the $N = 2$ theory), leading to supersymmetry breaking.

Thus, we have three cases in which :

i) There are no scalar moduli from ϕ_1 and ϕ_2;

ii) There is one complex modulus;

iii) Supersymmetry is broken.

The values of the angles are generally not relevant parameters for the low energy field theory, since if we change the angles the massless content does not change, except in particular cases where branes become parallel. The angles do, however, affect the massive modes of the field theory. In the discussion below we will specialize in most cases to angles of either 0 or 90 degrees.

As in [1], it is interesting to examine what a configuration of D3-branes stretched between a NS 5-brane and a NS′ 5-brane looks like from the 5-brane point of view. Recall that in [1], a D3-brane ending on a 5-brane looked like a magnetic monopole. So, if we have N_c D3-branes stretched between the two 5-branes, from the point of view of each 5-brane we have a configuration of magnetic charge N_c under the $U(1)$ gauge field living on the 5-brane. However, in this case, unlike the case described in [1], bringing the NS 5-branes together in the 67 directions (in the field theory this means taking the gauge coupling to infinity and the Fayet-Iliopoulos term to zero) does not correspond to an enhanced $U(2)$

gauge symmetry, but instead to a theory which has charged matter under the two $U(1)$ groups living on the NS 5-branes. This charged matter arises from D-strings, in the same way that for intersecting D-branes we get charged matter from fundamental strings. It consists of one hypermultiplet with charges $(1, -1)$ and $(-1, 1)$ under $U(1) \times U(1)$. The extra scalars may serve as additional parameters for the strongly coupled 3D field theory. There is no simple interpretation of the moduli space of this configuration as a monopole moduli space of some gauge theory, as in [1].

2.3. Global symmetry

As mentioned in §2.1, two global symmetries that are apparent in the construction when all angles are chosen to be zero or 90 degrees are $SO(2)_{45}$ and $SO(2)_{89}$. The scalar components of the fields ϕ_1 and ϕ_2 described above obviously have charges $(2, 0)$ and $(0, 2)$ under these symmetries (in a convenient normalization), while the other bosonic fields arising from D3-D3 strings are uncharged. The fermionic fields arising from D3-D3 strings are the reduction of a 10D spinor, so they have charges $+1$ or -1 under both $U(1)$ symmetries. We can identify $SO(2)_{89}$ with the $U(1)_R$ symmetry of the 3D $N = 2$ theory, and then $SO(2)_{45}$ is a combination of this $U(1)_R$ symmetry with a global symmetry under which ϕ_1 and ϕ_2 are oppositely charged.

When D5-branes are also present, quarks arise from open strings between the D3-branes and the D5-branes. For a D5-brane in the 012789 directions, these strings have Dirichlet-Neumann (DN) boundary conditions in the x_6, x_7, x_8 and x_9 directions, and DD or NN boundary conditions in all other directions. Thus, the states from the NS sector of the open strings, which we identify as the bosons in the quark multiplets, will be spinors of $SO(4)_{6789}$, and they will be charged, in particular, under $SO(2)_{89}$, but not under $SO(2)_{45}$. The fermions, on the other hand, come from the R sector of the open strings, and they are uncharged under $SO(4)_{6789}$ (and, therefore, also under $SO(2)_{89}$), but they will be charged under $SO(2)_{45}$ (since they arise, in the RNS formalism, from the quantization of RR zero modes in these directions). For D5-branes in the 012457 directions (D$'$ branes), $SO(2)_{45}$ and $SO(2)_{89}$ will be interchanged in this discussion.

In the construction of [1], there was an additional $U(2)$ global symmetry associated with coincident NS 5-branes (which was just the gauge symmetry of these 5-branes), that was broken to $U(1)^2$ for finite gauge coupling, but this does not appear in our case, as described at the end of the last section.

2.4. U(1) gauge theory with N_f flavors

We begin our analysis of 3D $N = 2$ gauge theories by considering a $U(1)$ theory with N_f flavors. The $U(1)$ gauge theory arises by considering a D3-brane stretched between two NS 5-branes, which are generally described by some equation $a_j z_1 + b_j z_2 = c_j$ ($j = 1, 2$) in the 4589 complex plane. The quark flavors arise from N_f D5-branes, which are similarly described by parameters $\tilde{a}_i, \tilde{b}_i, \tilde{c}_i$ ($i = 1, \cdots, N_f$), and by additional parameters z_i and $\tilde{m}_i$ corresponding to their x_6 and x_3 positions (we will assume $\tilde{m}_i = 0$ in the discussion below).

We would like to write down a superpotential describing this theory, including the quarks and the massless and massive ϕ_i fields. One term that obviously exists in such a superpotential is the coupling of the quarks to the adjoint fields, of the form

$$W = \sum_{i=1}^{N_f} (\tilde{a}_i \phi_1 + \tilde{b}_i \phi_2 - \tilde{c}_i) \tilde{Q}^i Q_i, \tag{2.7}$$

which is determined by the local $N = 4$ supersymmetry of the D3-D5 system (we normalize $\tilde{a}_i^2 + \tilde{b}_i^2 = 1$); the $\tilde{c}_i$ parameters have the obvious interpretation as quark masses, and we will set them (as well as the c_j) to zero in the discussion below (they can always be added later). However, it is clear that it is not always possible to include all the interactions in a superpotential. For instance, in the $N = 4$ theory of [1], the boundary conditions give a mass to ϕ_2 and ϕ_3, but it is impossible to write down a superpotential that incorporates this in an $N = 4$ invariant way (or even in a way which is invariant under the global symmetries, including the shift symmetry of the complex part of ϕ_3). This is not too surprising since ϕ_3 is in some sense actually a dual vector multiplet, so we would not expect to be able to write all its interactions in a superpotential. In any case, it is clear that in the configuration described above, the boundary condition on one side makes (in some way) $a_1 \phi_1 + b_1 \phi_2$ and ϕ_3 massive, while the boundary condition on the other side makes $a_2 \phi_1 + b_2 \phi_2$ and ϕ_3 massive. Since we do not know how to write down these mass terms explicitly, we cannot explicitly integrate out the massive fields and write down the low energy effective theory. However, we will assume that some consistent procedure for doing this exists, which will naturally give rise (starting from (2.7) and mass terms for the ϕ's) to quartic interactions between the quarks, and we will determine the low-energy superpotentials to fit the global symmetries and the moduli spaces we find in the various cases.

To simplify the discussion, we will assume that the two NS 5-branes are at zero and 90 degrees, i.e. they are a NS and NS' brane (the generalization of our results to other cases should be straightforward). In this case the first boundary condition gives a mass to ϕ_2 and ϕ_3, and the second to ϕ_1 and ϕ_3. While we cannot describe these mass terms by a superpotential, a superpotential of the form $\mu\phi_1\phi_2$ is consistent with the global symmetries described above, so we will assume that such a term is formed from the "boundary mass terms" when integrating out ϕ_3. This assumption will pass several consistency checks, as described below. Since we expect the masses of ϕ_1 and ϕ_2 to be of order g^2, we expect μ also to be of this order[2]. Of course, there is really an infinite tower of states corresponding to ϕ_1 and ϕ_2 arising in the dimensional reduction from the 4D theory to the 3D theory, but it turns out that (at least for our current purposes) using the naive mass term $\mu\phi_1\phi_2$ will suffice.

Let us begin with the simplest case of $N_f = 1$ (the $N_f = 0$ theory is obviously trivial, with just one massless vector multiplet). Adding the superpotentials we described above, we find for this theory

$$W = \mu\phi_1\phi_2 + (\tilde{a}_1\phi_1 + \tilde{b}_1\phi_2)Q\tilde{Q}. \tag{2.8}$$

Now, we can integrate out ϕ_1 and ϕ_2. If $\tilde{a}_1$ or $\tilde{b}_1$ are zero, so that the D-brane is oriented at zero or 90 degrees, this leaves no superpotential in the low-energy theory, which is just the $U(1)$ $N_f = 1$ gauge theory discussed in [14], which has a one dimensional Higgs branch (as well as Coulomb branches). However, at different angles, we find a low-energy superpotential of the form $W \sim \frac{\tilde{a}_1\tilde{b}_1}{\mu}(Q\tilde{Q})^2$, which lifts the Higgs branch. Note that in these cases the $SO(2)_{45}$ and $SO(2)_{89}$ symmetries are explicitly broken, so this superpotential is consistent with the global symmetries. The parameter in front of the quartic term is dimensionless, corresponding to g^2/μ if we had kept the dependence of the superpotential on the gauge coupling, and we expect it to be of order one for a generic angle. Comparing these results to what we see in the brane picture, we find an exact agreement. Only when the D5-brane is parallel to either the NS brane or the NS' brane (in the 45-89 plane) is there a flat direction corresponding to splitting the D3-brane at the D5-brane, which we identify with the Higgs branch of the field theory. Note that, in the full theory including

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[2] Note that in [6] it was assumed that the masses of the ϕ's were proportional to $\tan(\theta)$, where θ was the angle between the two NS 5-branes, and this passed some consistency checks. We expect this to be true only for small angles, when the mass is much smaller than the scale μ discussed here.

the massive fields, the field parametrizing this branch is really not just $M = Q\tilde{Q}$, but in fact a combination of this field and the chiral superfield corresponding to the motion of the D3-brane in the direction it moves along in the Higgs branch. This will be the case in all the examples described here.

Things become slightly more complicated when additional D-branes are added. For simplicity we will assume from here on that all D-branes are also oriented at zero or 90 degrees. This is the case where the Higgs branches have the maximal dimension, and the generalization to arbitrary angles should again be straightforward. For $N_f = 2$ there are now four different possibilities, depending on the orientation of the branes – in an obvious notation, corresponding to the ordering of the branes in x_6, they are NS-D-D-NS′, NS-D-D′-NS′, NS-D′-D-NS′ and NS-D′-D′-NS′. In the first case, the naive superpotential is

$$W = \mu\phi_1\phi_2 + \phi_1(Q_1\tilde{Q}^1 + Q_2\tilde{Q}^2), \tag{2.9}$$

and integrating out the massive fields leaves no low-energy superpotential. This is consistent with the brane moduli space in this case, as discussed in [13,14]. The fourth case is related to this one by an obvious rotation and reflection, and behaves in the same way. In the second and third cases, if we choose the first quark to come from the D brane and the second quark to come from the D′ brane, we would write a superpotential

$$W = \mu\phi_1\phi_2 + \phi_1Q_1\tilde{Q}^1 + \phi_2Q_2\tilde{Q}^2 \tag{2.10}$$

to describe both cases. Integrating out ϕ_1 and ϕ_2, we find $W \sim \frac{1}{\mu}Q_1\tilde{Q}^1Q_2\tilde{Q}^2$, and we can analyze the field theory with this superpotential by the methods used in [14]. We find that the moduli space of this theory consists of 6 one dimensional branches intersecting at the origin of moduli space, each of which is parametrized by one of $V_+, V_-, M_1^1, M_2^1, M_1^2$ and M_2^2 (where $M_i^j = Q_i\tilde{Q}^j$). This moduli space is actually the same (up to the quantum splitting of the Coulomb branch which is not visible in the brane picture) as that of the NS-D-D′-NS′ configuration. The Coulomb branch corresponds to a D3-brane connecting the NS and NS′ branes (free to move in the x_3 direction), the M_1^1 branch corresponds to splitting the D3-brane at the D brane (and then the D-NS′ piece of it is free to move in x_{89}), the M_2^2 branch corresponds to splitting the D3-brane at the D′ brane (and then the NS-D′ piece of it is free to move in x_{45}), while the M_1^2 and M_2^1 branches correspond to splitting the D3-brane twice, so that its middle part is free to move in x_7. Naively the double splitting corresponds to a single branch, but since this branch is identified with the Coulomb branch

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of the mirror theory [4,1,13,14] it is clear that it should be split in the quantum theory corresponding to this brane configuration (as described in [14]). Therefore, the NS-D-D'-NS' configuration is consistently described by the superpotential (2.10). However, this superpotential does not consistently describe the NS-D'-D-NS' configuration, in which a double splitting of the D3-brane leads to a three dimensional branch. In fact, this configuration is consistently described by the theory with no superpotential. Note that the only difference between these configurations is in the relative x_6 positions of the D-branes, which were called "hidden parameters" in [1] (and did not affect the low-energy field theory there). In the configurations of the type we discuss, it is clear that these parameters (or at least their signs) do affect the low-energy field theory, since they change the Higgs branches as discussed above. Since we do not know how to include the effects of these hidden parameters on the low-energy theory, we will again have to be content with guessing the results. In the $N_f = 2$ case, a consistent assumption seems to be that the quartic superpotential is generated for one sign of the x_6 distance between the D and D' 5-branes, corresponding to the NS-D-D'-NS' configuration, but no superpotential is generated for the other sign.

The results for other values of N_f are similar to the $N_f = 2$ case, and depend again on the ordering of the D and D' branes. When all D5-branes have the same orientation, no superpotential is generated (no superpotential is consistent with the global symmetries in this case), and this is consistent with the moduli space of the brane construction. When some of the branes are D (leading to quarks $Q_i, \tilde{Q}^i$) and some are D' (leading to quarks $q_j, \tilde{q}^j$), superpotentials of the form $W \sim Q_i \tilde{Q}^i q_j \tilde{q}^j$ may be generated as above (they are consistent with the global symmetries). For instance, the NS-D^n-(D')^m-NS' configuration is consistently described by a superpotential of the form $W \sim (\sum_{i=1}^n Q_i \tilde{Q}^i)(\sum_{j=1}^m q_j \tilde{q}^j)$, while the NS-(D')^m-D^n-NS' configuration is consistently described by $W = 0$. Note that these superpotentials are consistent with the $SU(n)$ and $SU(m)$ global symmetries that appear in the brane picture when we take all the D branes and all the D' branes together (the chiral global symmetries of these configurations will be discussed in §2.8). In other orderings of the branes, we can no longer take the D branes and D' branes together without interchanging them (which is a phase transition we do not know how to analyze), and the superpotentials no longer respect these symmetries. Since D3-branes between two D branes or between two D' branes have two chiral superfields describing their position, while D3-branes between a D brane and a D' brane have only one, it is clear that the maximum dimension of the Higgs branch is obtained in the NS-(D')^m-D^n-NS' configuration, while

smaller Higgs branches arise when the D and D′ branes are interposed. For each brane configuration we can write down some quartic superpotential that gives the correct moduli space, but the rules for doing so are not clear since we do not know the effect of the "hidden" x_6 parameters on the low-energy field theory.

The dependence of the superpotential on the x_6 ordering of the branes is consistent with the fact that the branes intersect each other when we move them past each other in the x_6 direction, so a non-trivial phase transition may occur. However, we can also take the D-branes around each other by moving them in the x_3 direction, which corresponds to giving a real mass to the quark before exchanging the branes. Obviously, once the quarks have a real mass (relative to the position of the D3-brane), they cannot obtain VEVs, so the superpotentials we write are no longer meaningful in this case. However, there should still be some continuous interpolation between the different superpotentials we write upon adding (and then removing) real masses for the quarks and shifting the branes in x_6. Similar problems arise when exchanging D and NS′ branes in the x_6 direction (also there a phase transition may naively be avoided after giving the quarks a mass), and they will be discussed in §2.9.

2.5. $U(N_c)$ gauge theories

The discussion of $U(N_c)$ theories with general angles for the D-branes is similar to the previous discussion, but with some important differences. Generally, we expect to get also in these theories quartic superpotentials for the quarks, and we expect that after taking these into account the field theory moduli spaces will agree with the brane construction. However, for $N_c > 1$, the global symmetries no longer uniquely determine the form of the superpotentials, even in the simplest cases.

An important difference between the $U(1)$ case and the non-Abelian cases is the existence of Euclidean-string-instanton configurations, which can lift part of the moduli space. For example, when two D3-branes are stretched between a NS and a NS′ 5-brane with no D5-branes between them, a superpotential is generated that drives these branes apart[3], corresponding to the Affleck-Harvey-Witten instanton-generated superpotential in field theory [15,13,14]. In this case the instanton effects in the brane picture agree with

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[3] The boundaries of the D3-branes in the 5-branes look like 4D $N = 2$ monopoles, but the presence of the other 5-branes breaks the $N = 2$ to $N = 1$, so there is no longer any reason for the forces between these monopoles to cancel.

our low-energy field theory description. However, another case where a superpotential appears to be generated in the brane picture is when D3-branes are stretched between D and D′ 5-branes. Since this is the $SL(2,\mathbb{Z})$ dual of the previous case, we expect Euclidean fundamental strings to generate a superpotential in this case that will drive the D3-branes apart and lift the corresponding Higgs branches. From the point of view of the mirror theory, this superpotential is the standard Affleck-Harvey-Witten superpotential, but in the original theory it is an instanton effect lifting part of the Higgs branch, which we do not know how to describe in the low-energy theory (it depends on the "magnetic gauge coupling" in the language of [1], which we do not know how to identify in the conventional field theory description). Surprisingly, the low-energy field theory superpotentials are consistent with simply ignoring those branches of the brane picture which are apparently lifted by such instanton effects, as we describe below.

For example, let us analyze in detail the $U(2)$ theory with $N_f = 2$. If the two D-branes are parallel, the arguments above (and the global symmetries) suggest that there is no superpotential, and the brane moduli space is consistent with this assumption. In the NS-D-D′-NS′ and NS-D′-D-NS′ configurations, we again seem to have a superpotential of the form

$$W = \mu\mathrm{tr}(\phi_1\phi_2) + Q_1\phi_1\tilde{Q}^1 + Q_2\phi_2\tilde{Q}^2, \tag{2.11}$$

where ϕ_1 and ϕ_2 are now $U(2)$-adjoint matrices. Note that the D3-brane worldvolume theory now includes also an interaction of the form $W \sim \mathrm{tr}([\phi_1,\phi_2]\phi_3)$, whose effect on the low-energy dynamics is not clear. In any case, integrating out ϕ_1 and ϕ_2 in (2.11), we find a superpotential of the form $W \sim M_1^2 M_2^1$. Unlike the $U(1)$ case, however, this is not the most general superpotential consistent with the global symmetries, and a term of the form $W \sim M_1^1 M_2^2$ might also appear.

As we found for the $U(1)$ case, the moduli space of the NS-D-D′-NS′ configuration is consistent with the presence of superpotentials of this form. Using the fact (required for consistency [1]) that there are no supersymmetric s-configurations (in which more than one D3-brane is stretched between the same D and NS branes, or between the same D′ and NS′ branes), the most general Higgs branch of this configuration, which is two dimensional, appears in figure 2(a). We can identify this branch (assuming the superpotential is just $W \sim M_1^2 M_2^1$) with the Higgs branch of the field theory, where M_1^1 and M_2^2 are non-zero. We can also analyze the Coulomb branches and the mixed branches, by adding the superpotential which consistently describes the quantum moduli space of this theory, of the
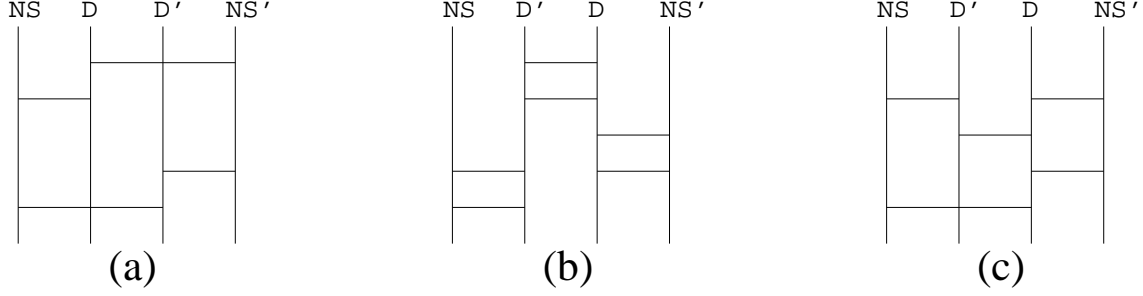
Figure 2: Some Higgs branches of $U(2)$ theories with two flavors, in a schematic representation (the horizontal axis is x_6, but the vertical axis corresponds to no particular coordinate) : (a) The most general Higgs branch of the NS-D-D′-NS′ configuration, (b) A Higgs branch lifted by instantons in the NS-D′-D-NS′ configuration, (c) An unlifted Higgs branch in the NS-D′-D-NS′ configuration.

form $W = V_+V_-(M_1^1 M_2^2 - M_2^1 M_1^2)$ [14]. For instance, the field theory has a branch where M_1^1 and V_- are both non-zero. In the brane picture, this corresponds to splitting one of the 3-branes twice, and leaving the other 3-brane continuous (as before, a quantum splitting which is not visible in the naive brane picture means that the same type of configuration, with different brane positions, describes also the branch where M_2^2 and V_- are non-zero).

The analysis of the NS-D′-D-NS′ configuration is more complicated here, since some of its branches, like the one depicted in figure 2(b), which is naively six dimensional, are lifted by instanton effects. However, the remaining branches seem to correspond to those of the field theory with $W = 0$, as we found also in the $U(1)$ case. For example, the unlifted Higgs branch depicted in figure 2(c) is four dimensional, which is the same dimension we find in the field theory. Higgs branches with an unbroken $U(1)$ are three dimensional in the brane configuration as well as in the field theory.

Similar considerations apply also for higher N_c and N_f. Consider, for instance, higher values of N_c with $N_f = 2$. For $N_c > 3$ there are no unlifted branches in the NS-D′-D-NS′ configuration, consistent with the assumption of $W = 0$ in this case, since the corresponding field theory has no supersymmetric vacua either [14]. For $N_c = 3$ there are unlifted five dimensional Higgs-Coulomb branches (with D3-brane segments connecting, for instance, NS-NS′, NS-D′, D′-D, D-NS′, NS-D′ and D′-NS′), in agreement with the field theory constraint $V_+V_- \det(M) = 1$ [14]. In the NS-D-D′-NS′ configuration of $U(3)$ with $N_f = 2$ we find only a three dimensional Higgs-Coulomb branch, in agreement with the constraint and the additional superpotential $W \sim M_1^2 M_2^1$ which sets $M_1^2 = M_2^1 = 0$. For higher values of N_f, as in the $U(1)$ case, more general superpotentials arise, depending on

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the ordering of the branes. For each ordering there seems to be a particular superpotential, consistent with the global symmetries, that correctly gives the brane moduli space.

2.6. Mirror symmetry

The configurations we discuss here can be used to construct mirror symmetries of three dimensional gauge theories [4], in the same way as in [1] for $N = 4$ theories and in [13,14] for $N = 2$ theories. However, we can also derive these "new" mirror symmetries directly from the known mirror symmetries of the $N = 2$ SQCD theory. Since the theories we discuss differ from the standard SQCD theories just by having an additional superpotential, their mirror is just the mirror of the standard SQCD theory, plus a superpotential which is the image of the additional superpotential under the mirror transformation. This is analogous to the relation between the $N = 4$ mirror symmetry and the $N = 2$ mirror symmetry [14].

For example, let us look at the $U(1)$ theory with $N_f = 2$ in the NS-D-D′-NS′ configuration, which we claimed corresponded to $W \sim Q_1 \tilde{Q}^1 Q_2 \tilde{Q}^2$. Performing the mirror transformation of $SL(2, \mathbb{Z})$ followed by rotations and exchanging orders of branes, as in [1], we come back to exactly the same configuration, so we claim this theory is self-mirror (like the corresponding $N = 4$ theory). However, this may also be seen directly from the known $N = 2$ mirror for the theory with no superpotential [13,14]. This mirror is a $U(1)$ theory with two flavors $q_i, \tilde{q}^i$, two singlets S_i, and a superpotential $W = S_1 q_1 \tilde{q}^1 + S_2 q_2 \tilde{q}^2$. Since the S_i are identified with $Q_i \tilde{Q}^i$ [14], the additional superpotential in the mirror theory is $W \sim S_1 S_2$, and then integrating out S_1 and S_2 we indeed get the same quartic superpotential as in the original theory.

2.7. Branes and Superpotentials in 4D $N = 1$ Supersymmetric Gauge Theories

Much of the discussion of this section can be generalized also to brane constructions of 4D $N = 1$ field theories, which are different from the constructions described above only in the fact that all the D-branes stretch along the x_3 direction as well. One obvious difference is that the vector multiplet no longer contains any scalars, so there are no longer any Coulomb branches, but the analysis of the Higgs branches described above is essentially the same. In particular, also in brane constructions of 4D $N = 1$ theories with D6-branes in different orientations we expect to get quartic superpotentials, as described above. In the 4D theory, the coefficient of this superpotential carries (in the UV) a negative mass dimension, and we expect it to be of the order of one over the string scale. However, the

presence of the superpotential still lifts some of the Higgs branches, and the field theory with the superpotential still describes correctly the brane construction.

There are two types of instanton effects in the brane constructions of the 4D $N = 1$ theory, related to the two types of instanton effects in 3D $N = 2$ theories described above. First, there are the standard gauge theory instantons, which (in the zero size limit) look like D0-branes inside the D4-branes, and which are expected to generate superpotentials lifting the Higgs branches in some cases (as in [16]). Then, there are the "Higgs branch instantons" similar to the ones which appeared in 3D. When two D4-branes stretch between D and D' 6-branes, with no other branes nearby, the effect of an Euclidean string stretched between them is expected to lift this branch, in a way which we cannot describe in the original field theory, since it depends on parameters which are not visible there (in the 4D case this instanton is not related by any duality to the standard instantons). As in the 3D case discussed above, the lifting of these branches is consistent with the fact that they do not appear in the corresponding field theories. The full analysis of the brane configurations is, however, complicated by the fact that the standard 4D instanton effects are not easily visible in these configurations.

As we did for mirror symmetry in the previous section, we can also generalize the constructions of Seiberg dualities [5] from branes [2] (see also [17,18,6,19,20]) to the configurations with D-branes at different angles. However, again, all we get is a deformation of the known dualities by an operator quartic in the quarks of the original theory, which we can identify also in the dual theory. For instance, beginning with the NS-D^n-(D')^m-NS' configuration, which we claimed was described by $W \sim (Q_i \tilde{q}^j)(q_j \tilde{Q}^i)$ (where i goes over the D-quarks and j goes over the D'-quarks), we can move all D branes to the left and all D' branes to the right without changing the low-energy theory, and then exchange the NS and NS' 5-branes as in [1,2]. In the brane construction, after the exchange we find an $SU(N_f - N_c)$ theory with $n + m$ flavors, with n^2 mesons coupling to n of the dual quarks and m^2 mesons coupling to the other m dual quarks. This is exactly what we find also just by adding the operator corresponding to the superpotential W above to the standard Seiberg dual [5], since it just gives a mass to the "off-diagonal" mesons. Integrating out these mesons we find a quartic superpotential also for the dual quarks, which we expect to arise in the dual brane configuration in the same way discussed above, and which lifts the dual off-diagonal Higgs branches (that do not exist in the original theory). The same discussion applies also to the 3D $N = 2$ configurations using the duality described in [21] (see also [22]). The microscopic definition of this duality is not yet known, but it correctly matches the effective IR theories describing the "electric" and "magnetic" theories.

2.8. Chiral symmetry

It was conjectured in [6], in the context of 4D $N = 1$ gauge theories realized through type IIA superstrings, that non-Abelian chiral symmetry is manifested in a particular configuration of the branes[4]. This happens when a D6-brane parallel to a NS 5-brane coincide. The gauge symmetry on the worldvolume mutual to both types of branes, when there are n coincident D6-branes, seems to be enhanced from $SU(n)_V$ to $SU(n)_L \times SU(n)_R$ when the position of the D6-branes coincides with the position of the NS brane. The same construction obviously holds also for the 3D $N = 2$ theories discussed in this paper, when D and NS′ 5-branes (or D′ and NS 5-branes) come together. In this case the D5-branes can actually split along the NS 5-brane, as discussed in the next section, and the implications of this for chiral symmetry are discussed at the end of section 4.

Naively, we would not expect to find the non-Abelian chiral symmetry in the brane constructions, even though it exists in the low-energy field theory. This is because, as described above in detail, the massive ϕ fields couple to the quarks in a way which breaks the chiral symmetry. In particular, the quartic superpotentials we described above, which we get when integrating out these ϕ fields, are obviously not invariant under the chiral symmetry. However, in the particular configurations where all the D′ branes are to the left of all the D branes, we found that no such quartic superpotential exists. These are exactly the configurations where we can move (in x_6) all the D′ branes to intersect with the NS branes, and all the D branes to intersect the NS′ branes, without encountering any phase transitions, and thus, this is the only case where the arguments of [6] for chiral symmetry hold. Therefore, the superpotentials we find are consistent with the conjecture of [6]. However, it is still not clear why the quartic superpotentials breaking the chiral symmetry are not generated in these configurations, or, equivalently, why the apparent coupling of the ϕ fields to the quarks no longer breaks the chiral symmetry. The consistency of the conjecture of [6] seems to require that either the $\phi Q \tilde{Q}$ coupling goes away when the "flavor" D-branes intersect the NS 5-branes, or that the mass of ϕ goes to infinity in this limit. Both assumptions are consistent with our results here, but it is not clear why they should be true. This issue is obviously related to the phase transition when a D brane passes a NS′ brane in x_6, since the chiral symmetry is claimed to be manifest at the point of this phase transition. This phase transition is discussed in the next section.

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[4] The axial $U(1)$ is usually directly visible in the brane constructions – when all D-branes have the same orientation it is just the difference between $SO(2)_{45}$ and $SO(2)_{89}$.

2.9. Other effects of "hidden parameters"

In §2.4 we saw that the relative x_6 positions of the D5-branes, which were "hidden parameters" in the $N = 4$ theory discussed in [1], actually have an effect on the low-energy field theory in some cases. In the $N = 2$ configurations we discuss here, there is also another "hidden parameter" that could affect the low-energy theory, which is the relative x_6 position of D and NS′ branes (or of D′ and NS branes). Recall that in [1] it was discovered that moving a D brane through a NS brane led to a phase transition in which a new D3-brane was generated, enabling the low-energy field theory to stay the same after the transition. Thus, the relative x_6 positions of D and NS branes (or of D′ and NS′ branes) do not affect the low-energy field theory.

However, for this phase transition to occur it was crucial that the D and NS branes had to intersect when they were interchanged, and this is no longer true for D and NS′ branes (though it is true for any other angle, for which the branes are not parallel in the 45-89 complex plane). D and NS′ branes can be interchanged by moving them around each other in the x_4 or x_5 directions, and then it seems clear that no phase transition can occur when they are interchanged, since they do not have to pass near each other. There is also no linking number argument for a phase transition in this case. Thus, the apparent conclusion would be that there is no phase transition when D and NS′ branes are interchanged, and then their relative x_6 position obviously affects the low-energy dynamics (since we have a massless quark when the D is between the NS and the NS′ but not otherwise).

However, this conclusion might be too naive. When we move the branes around each other as described above, we are giving the quarks arising from the D5-branes a mass. If we want the D5-brane and the NS 5-brane to really go around each other smoothly, the distance between them should be larger than the string scale, but in this case the quark mass is also very large, and it can no longer be included in the low-energy effective theory (since there are many other states whose mass is of the order of the string scale in the brane construction). So, we cannot really follow what happens to the quarks in the low-energy field theory by taking the branes around each other, and a phase transition might still occur if the branes are interchanged at a distance smaller than the string scale. Unfortunately, it does not seem possible to analyze this transition by string theory methods, nor by the methods used in [17] to study the phase transition of [1]. The issue of this phase
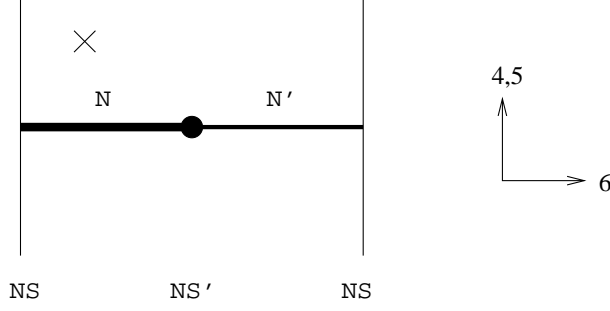
Figure 3: A phase transition through "hidden parameters". There are N D3-branes stretched between NS and NS$'$ 5-branes, and N' D3-branes stretched between NS$'$ and NS 5-branes. A D 5-brane (denoted by "X") can move in the x_6 direction to give different massless matter contents.

transition, and in general of the x_6 dependence of the low-energy field theory as described above, deserves further investigation.

As a specific example let us consider a configuration of three NS 5-branes as depicted in figure 3. The two external 5-branes are NS and the 5-brane in the middle is NS$'$. There are N D3-branes between the left NS brane and the NS$'$ brane, and N' D3-branes between the right NS brane and the NS$'$ brane. There is also a D 5-brane located at arbitrary positions in $x_{3,4,5,6}$. Denote the x_6 position of the D 5-brane by z and the x_6 position of the NS$'$ brane by t. The 3D gauge group is $U(N) \times U(N')$. The D brane gives rise to a (generally massive) quark (and perhaps other states as well). Moving the D 5-brane in the 3 and 45 directions corresponds to changing the bare real and complex masses, respectively, of the quark. In particular, when the D brane touches the D3-branes, there is a massless quark. The matter content depends, however, on the ordering of z and t. For $z < t$ we have a massless quark in the $(N, 1)$ representation of the gauge group while for $z > t$ we have a massless quark in the $(1, N')$ representation of the gauge group. There is a smooth process which interpolates between the two regions. Starting with a massless quark in the first region, we can move the D5-brane in the 45 directions, change the x_6 ordering and move back to get a massless quark in the second region. There is no D3-brane created in the process as the NS$'$ and D branes can go around each other, but the massless matter content is obviously changed. From a field theory point of view such a process, of a change in the representation of a field, would be very surprising, but, as discussed above, it seems that we cannot really discuss such processes in the low-energy field theory. Presumably, this type of process indicates that there are massive states in the theory whose mass depends on the "hidden" x_6 positions.

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3. "Polymeric" 5-branes and Five Dimensional Field Theories

In this section we will construct five dimensional gauge theories, and other five dimensional superconformal field theories, using D5-branes and NS 5-branes in type IIB string theory. The construction we use will be a generalization of the construction of [1] in 3 dimensions, and of [3] in 4 dimensions, but we will use slightly different conventions for reasons that will become clear when we relate our results here to three dimensional gauge theories in section 4.
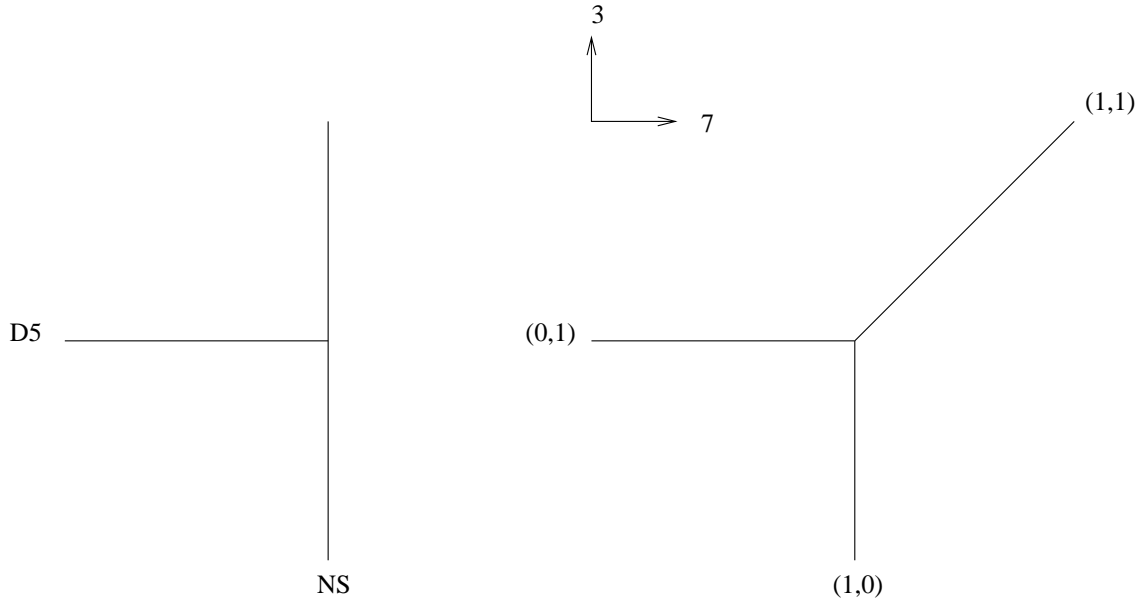
Figure 4: A D5-brane which ends on a NS 5-brane. The left side describes the naive configuration, and the right side the correct configuration, which implements conservation of charge at the vertex.

Consider a NS 5-brane along the 012389 coordinates and a D5-brane along the 012789 coordinates which ends on a it, as drawn on the left side of figure 4 (this is a naive description of the situation which will be improved below). This system is T-dual to any D-brane ending on a NS 5-brane, like in the configurations studied in [1]. Thus, it is easy to check that the supersymmetry is broken to 1/4 of the original supersymmetries. This means that we are dealing with minimal ($N = 1$) supersymmetric six dimensional gauge theories, or five dimensional gauge theories if we make some of the 5-branes finite (as we did for the D3-branes above). These theories have an $SU(2)_R$ global symmetry, which is a double cover of the $SO(3)$ rotation group in the x_4, x_5 and x_6 directions. The boundary of the D5-brane is a 4-brane which propagates in $5+1$ dimensions, and thus has the behavior

of a particle in $1+1$ dimensions. There are four scalars which parametrize the position of the NS 5-brane in 4567 space, but it is clear from symmetry arguments that only the x_7 coordinate is affected by the presence of the D5-brane, and it will depend on the position of the D5-brane in the x_3 direction (the analysis here is a generalization of the analysis of [3] for D4-branes ending on NS 5-branes). For large x_3, the classical equation which governs this dependence is the Laplace equation in one spatial dimension :

$$\nabla^2 x_7 = \delta(x_3),\tag{3.1}$$

where $x_3 = 0$ is the position of the D5-brane inside the NS brane. The general solution of this equation is

$$x_7 = \frac{1}{2}|x_3| + cx_3 + d,\tag{3.2}$$

where c and d are constants. We will choose the constants so that for large negative values of x_3, we have a standard NS(012389) 5-brane at $x_7 = 0$ – this determines $c = \frac{1}{2}$ and $d = 0$. At this point it seems like the supersymmetry is broken, since we started with a NS 5-brane along the x_3 coordinate, and we end up (for $x_3 > 0$) with a brane which is diagonal in the $x_3 - x_7$ plane. How can this be possible? Recall that in fact, in the type IIB string theory, there are bound states of p NS 5-branes and q D5-branes (for p and q relatively prime), called (p,q) 5-branes, which behave just like standard 5-branes (and, in fact, are related to them by $SL(2,\mathbb{Z})$ U-duality transformations). Charge conservation does not really allow a D5-brane to end on a NS 5-brane – instead, at the intersection point the two 5-branes merge together to form a $(1,1)$ 5-brane, as drawn on the right side of figure 4. Charge conservation does not constrain the configuration any further. However, if we want the "new" brane coming out of the intersection point not to break the supersymmetry any further, it must be oriented at a 45 degree angle in the $x_3 - x_7$ plane, exactly as (3.2) implies. In the same way general vertices of (p,q) 5-branes may be drawn, subject to charge conservation (a similar discussion for (p,q) strings appears in [23]), where the angle θ of each (p,q) 5-brane in the $x_3 - x_7$ plane satisfies $\tan(\theta) = p/q$ so as to preserve the remaining supersymmetry.

Naively, we may expect corrections to equation (3.2), but there are no objects which can contribute to the metric of the configuration. Any corrections would also break the remaining supersymmetry. This is consistent with the expectations from field theory (when we use these configurations to construct 5D field theories, as described below), since in five dimensions there are no instanton corrections to the metric.

We can easily extend this analysis to cases in which there is more than one D5-brane ending on a (asymptotically) NS 5-brane from both sides. Let a_i, $i = 1, \ldots, m$ be the x_3 positions of the D5-branes ending from the left on the NS 5-brane, and b_j, $j = 1, \ldots, n$ be the x_3 positions of D5-branes ending from the right on the NS 5-brane. The solution of the generalization of (3.1) then takes the form

$$x_7 = \frac{1}{2}(\sum_{i=1}^{m} |x_3 - a_i| - \sum_{j=1}^{n} |x_3 - b_j|) + cx_3 + d. \tag{3.3}$$

For $c = \frac{m-n}{2}$ and $d = 0$ this equation has the interpretation of a 5-brane which starts far away as a NS 5-brane, and changes its charge, and its $x_3 - x_7$ angle, in places where a D5-brane ends on it. The change of charge is dictated by a conservation law which states that the sum of charges be zero at each vertex point, while the angle is determined by supersymmetry (or by (3.3)). A condition for a NS brane to stay NS for large x_3 is that $m = n$. In all other cases the 5-brane looks different at the far ends. The piecewise linear function (3.3) is reminiscent of the five dimensional gauge coupling functions described in [7,8,9,10], and it is in fact closely related to them, as described in the next section.

3.1. Pure $SU(2)$ gauge theory

Next, we turn to the construction of five dimensional gauge theories using the branes. As in [1,3], a configuration of N_c parallel D5-branes of finite extent, stretched between NS 5-branes, is expected to give a $U(N_c)$ (or $SU(N_c)$) 5D $N = 1$ supersymmetric gauge theory. For a given configuration of branes there can be two types of deformations : one type which does not change the asymptotic form of the 5-branes, and a second type which changes the asymptotic form of the 5-branes. As in [3], we interpret the first type of deformation as a change in the dynamical moduli of the system, while the second type is interpreted as a change in the parameters which define the field theory.

Let us begin with the simplest configuration of two parallel D5-branes. According to the previous discussion, this configuration actually looks like the left hand side of figure 5(a), where we chose (arbitrarily) specific orientations for the outgoing branes. Apriori this seems to represent a $U(2)$ gauge theory, and one might expect also additional contributions from the finite segments of the NS 5-branes, which also correspond to 5D theories at low energies. However, as in [3], it is easy to see that there is only one deformation of this configuration which does not change the asymptotic forms of the branes in figure 5, corresponding to going from figure 5(a) to 5(b). Thus, the configuration 5(a) has only one
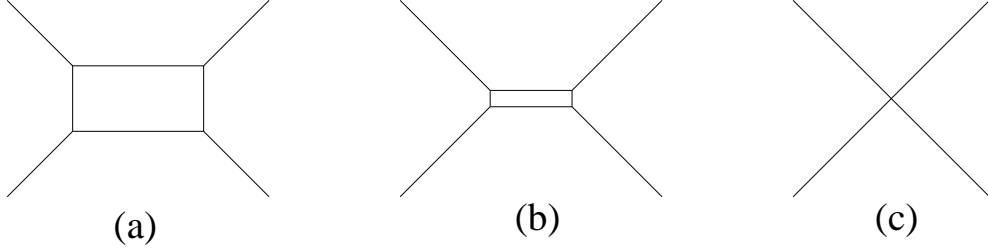
Figure 5: Pure $SU(2)$ gauge theory in five dimensions. Horizontal lines represent D5-branes, vertical lines represent NS 5-branes, and diagonal lines at an angle θ such that $\tan(\theta) = p/q$ represent (p, q) 5-branes. Figure (a) shows a generic point on the Coulomb branch, figure (b) shows a point near the origin of moduli space, and figure (c) corresponds to the strong coupling fixed point.

real massless scalar, which (using the 5D supersymmetry) necessarily corresponds to having just one massless vector multiplet (and no massless hypermultiplets). We will denote this scalar by ϕ; it can be chosen to correspond to the distance between the two D5-branes. We expect to find an unbroken $SU(2)$ gauge symmetry when the two D5-branes come together, as in 5(b). This is consistent with the obvious $\phi \to -\phi$ symmetry, which implies that the vector multiplet we see is indeed in the Cartan subalgebra of an $SU(2)$ gauge theory. Thus, we suggest that the configuration 5(a) corresponds to a generic point on the Coulomb branch of the 5D $N = 1$ pure $SU(2)$ gauge theory.

We can check this conjecture by examining various properties of this configuration. As in [1,3], it is natural to identify the bare gauge coupling $1/g_0^2$ with the length of the two D5-branes at the point in moduli space where they come together (up to a constant involving the string coupling). The Coulomb branch of the $SU(2)$ gauge theory has a prepotential of the (exact) form [7]

$$\mathcal{F} = \frac{1}{2g_0^2}\Phi^2 + \frac{c}{6}\Phi^3, \tag{3.4}$$

where $c = 16 - 2N_f$ arises at one loop. The masses of electrically charged BPS saturated states are proportional to ϕ, while those of magnetically charged BPS saturated states (which are strings in 5D) are proportional to $\frac{\partial \mathcal{F}}{\partial \phi}$. In the configuration of figure 5, the electrically charged particles (W bosons) arise from strings between the two D5-branes, so their mass is naturally proportional to ϕ. As in [1], the magnetic monopole may be identified with a D3-brane spanning the rectangle between the D and NS 5-branes (thus giving rise to 5D strings), since a D3-brane looks like a magnetic monopole from the point of view of any 5-brane it ends on. The tension of these strings is proportional to the area

of the rectangle, which is (up to constants) $\phi/g_0^2 + \phi^2$, in agreement with (3.4) (up to multiplicative redefinitions of g_0 and ϕ).

The $SU(2)$ gauge theory with no flavors is argued to have a non-trivial fixed point as $g_0 \to \infty$ [7]. In the brane construction, this corresponds to figure 5(c) (choosing $\phi = 0$). Thus, we claim that this fixed point is the same as the one corresponding to the intersection of $(1,1)$ and $(1,-1)$ 5-branes at 90 degrees (with five dimensions common to both branes). Note that this configuration is not equivalent in any way to a configuration of a NS 5-brane and a D5-brane intersecting at 90 degrees, which has no Coulomb branch.

The brane construction suggests several deformations of this fixed point (here we discuss changes of parameters of the 5D field theory, which correspond to the asymptotic locations/orientations of the 5-branes, as opposed to local changes which correspond to 5D fields such as ϕ).

The obvious deformation is increasing $1/g_0^2$, going from figure 5(c) to 5(b). This deformation is visible from the field theory point of view [7] and corresponds to turning on a finite $1/g_0^2$ in a $SU(2)$ gauge theory, which then flows to a trivial IR fixed point.

A deformation in the opposite direction, which might be called a continuation of the field theory to negative values of $1/g_0^2$, leads to a configuration which looks like a 90 degree rotation of figure 5(b). This looks like an $SU(2)$ gauge theory with coupling $|1/g_0^2|$, but now the $SU(2)$ gauge symmetry comes (at least naively) from the NS 5-branes instead of from the D5-branes.

There is one other deformation we can do at the infinite coupling fixed point without breaking the supersymmetry, which corresponds to moving the two branes which intersect there apart in the $x_{4,5,6}$ directions. After this deformation there are no longer any massless five dimensional fields. When the two 5-branes approach each other, the lowest lying 5D states naively correspond to membranes whose tension is proportional to the distance between the two 5-branes. These arise from D3-branes ending on both branes (these are the only branes which can do so). Thus, from this point of view the $SU(2)$ strong coupling fixed point has "tensionless membranes". In the construction of these theories via M theory on Calabi-Yau manifolds [8,9,24], such deformations correspond to blowing up 3-cycles (and the membranes arise from 5-branes wrapped around these 3-cycles)[5]. It may be possible to translate our constructions directly into those of [8,9,24] by translating 5-branes into geometric singularities as in [17].

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[5] The number of such 3-cycles in some particular cases was computed in [25].

In our construction of the $SU(2)$ theory we made an arbitrary choice of the orientation of the outgoing 5-branes. Instead of choosing $(1,1)$ and $(-1,1)$ 5-branes for large values of x_3, we could have chosen general (p_1, q_1) and (p_2, q_2) 5-branes, and most of the discussion above would have remained unchanged (at least as long as all the branes appearing have (p,q) relatively prime). The Coulomb branches corresponding to all these constructions are obviously isomorphic, but it is not clear to us if the strong coupling fixed points corresponding to these different constructions are the same or not (i.e. whether they have the same operators and OPEs). The theories might differ by irrelevant operators. Recall that, as discussed in [7] and reviewed in §3.4 below, the $SU(2)$ pure gauge theory is known to have two different possible strong coupling limits, called E_1 and $\tilde{E}_1$ (corresponding to different discrete "theta parameters" [9]). One property of the strong coupling fixed points which we can try to read off from the brane configuration is their global symmetries, though it is not clear if all the non-Abelian factors of the global symmetries must be manifest in this type of construction. As discussed in §3.2 below, the rank of the global symmetry for any choice of asymptotic branes is one, but generically there is no sign of the enhanced non-Abelian gauge symmetry that corresponds to the $E_1(= SU(2))$ fixed point [7].
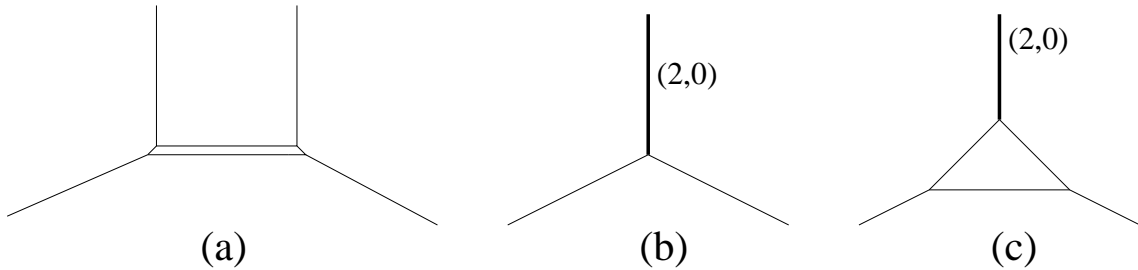

Figure 6: Another construction of the pure $SU(2)$ gauge theory. Figure (a) shows a point near the origin of the Coulomb branch, figure (b) shows the strong coupling fixed point, and figure (c) shows the Coulomb branch emanating from this strong coupling fixed point.

In one particular configuration, described in figure 6(a), where we choose both of the asymptotic branes for large x_3 to be NS 5-branes, we do see an enhanced $SU(2)$ global symmetry at the strong coupling fixed point, described by figure 6(b), where the two NS 5-branes overlap. It is not clear if this $SU(2)$ global symmetry exists (but is hidden) for other constructions of the strong coupling fixed point, or if the other constructions describe different fixed points. In the latter case our constructions give many new 5D $N = 1$ superconformal field theories, all of which have a one dimensional Coulomb branch similar to that of the $SU(2)$ pure gauge theory. Another difference between the brane

constructions with different asymptotic branes is that only in the original construction of figure 5 do we see the deformation corresponding to having membranes with small tensions. Again, this might indicate a difference between the different fixed points, or that this deformation is just not visible in the other brane constructions of the same fixed point. We have not been able to determine whether the various fixed points are the same or not.

3.2. General properties of 5D fixed points from branes

As discussed above, it seems possible to get five dimensional SCFTs from the low-energy theory at the intersection of several half 5-branes (each of which has some charges (p, q)). In this section we discuss the general properties of such fixed points which appear in the brane constructions.

The obvious generalization of the fixed points described in the previous section is to a configuration of n half 5-branes which all emanate from the same point. We claim that each such configuration corresponds to a five dimensional SCFT (though in some cases, like $n = 2$, this theory will be trivial). We will denote the charges of the half 5-branes, when they are all oriented towards the intersection point (i.e. their orientation 6-form is $dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_8 \wedge dx_9$ times a vector in the direction of the intersection point), by (p_i, q_i) $(i = 1, \cdots, n)$. Charge conservation obviously requires $\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i = 0$, and the angles of the branes are related to their charges by the condition of preserving the remaining supersymmetry. Note that these conditions are not $SL(2, \mathbb{Z})$ invariant (the S generator of $SL(2, \mathbb{Z})$ acts on them in the same way as a 90 degree rotation in the $x_3 - x_7$ plane, but there is nothing similar for the T generator), so we cannot do an $SL(2, \mathbb{Z})$ transformation taking, say, (p_1, q_1) to $(0, 1)$ without (generally) changing the fixed point theory. We will assume that all the (p_i, q_i) are relatively prime. A general 5-brane for which $(p, q) = m(p_0, q_0)$ may be viewed as a collection of m (p_0, q_0) 5-branes (and will be drawn as a wide line in our figures).

This construction defines (implicitly) the fixed point theory corresponding to particular values of (p_i, q_i). There are two obvious types of questions we can ask about these fixed points in the brane configurations. First, we can ask what are the Coulomb branches coming out of these fixed points, generalizing the $SU(2)$ Coulomb branch described in the previous section. These branches correspond to deformations of the 5-branes near the intersection point which do not change the asymptotic behavior of the branes. They may include several parallel D5-branes, as in the previous section, in which case we can interpret them (at least naively) as corresponding to a non-Abelian gauge theory arising

from these D5-branes. This will be discussed in the next section. There could also be Coulomb branches which have several parallel (p, q) branes, which we could also interpret in the same way, or branches which have no parallel branes at all. An example of the latter possibility is the strong coupling limit of the $SU(2)$ $N_f = 0$ theory (described in the previous section) with two asymptotic NS 5-branes, whose Coulomb branch is described in 6(c). In such cases we cannot interpret the W bosons as corresponding to particular strings between 5-branes, and they correspond to more general states (which, in principle, are combinations of states from all the 5-branes which have one finite direction). A general analysis of the Coulomb branch emanating from a particular fixed point (and, in particular, a computation of its dimension as a function of the charges (p_i, q_i)) should be possible, but we will not perform it here, and discuss only some specific cases.

The other properties of the fixed points which we can read off from the brane constructions are their deformations and global symmetries (which are related since some of the deformations may be thought of as turning on background global vector fields). There seem to be two types of deformations which do not break the supersymmetry. First, we can move the branes in the $x_3 - x_7$ plane without changing their orientation. The line corresponding to each (p_i, q_i) 5-brane is given by an equation of the form $p_i x_7 + q_i x_3 = c_i$ for some real number c_i, so naively we have n real parameters corresponding to the positions of the branes. However, if the branes are not all parallel (in which case there is no interesting 5D fixed point), two of these parameters correspond to setting the origin of the x_3 and x_7 coordinates, and another is set by the requirement that $\sum_{i=1}^n c_i = 0$. This requirement is obvious if all 5-branes are to intersect at one point, but it holds also in more general configurations which involve also finite 5-brane segments, since it holds at each intersection point separately. Thus, there are only $n - 3$ real parameters corresponding to deformations of the fixed point theories. These parameters are the scalar components of vector multiplets inside the 5-branes. Naively, there is (at least) a $U(1)^n$ gauge symmetry on the 5-branes, which may be interpreted as (part of) the global symmetry of the fixed points, but the discussion above shows that only a $U(1)^{n-3}$ subgroup of this acts non-trivially on the fixed points, and the deformations described above can be viewed as background vector fields for these global symmetries. When m of the 5-branes have the same (p_i, q_i) and c_i, the global symmetry is enhanced to (at least) $SU(m)$, but its rank always remains $n - 3$ (and the deformations correspond in general to the Cartan subalgebra of the global symmetry group).

The second type of deformation which is visible in the brane constructions corresponds to moving branes away in the x_4, x_5 or x_6 directions. Such a deformation appears whenever two of the branes have opposite (p_i, q_i) and c_i, and then we can join them together to one infinite (as opposed to semi-infinite) 5-brane and move them off. This type of deformation is charged under the $SU(2)_R$ global symmetry (which we identified with $SO(3)_{456}$), so it cannot be described by a background vector multiplet. Instead, its form is like that of a Fayet-Iliopoulos term, but its interpretation as such is not clear since the $U(1)$ factors of the 5D gauge group seem to be projected out, as described above. Note that deformations of this sort are generally possible only at the strong coupling fixed points, and not at generic points on the Coulomb branch, so it is not clear that they should have a field theory interpretation. When approaching the fixed point from the direction of a deformation of this sort, the 5D theory seems to have membranes whose tension goes to zero at the fixed point.

3.3. Generalizations to $SU(N_c)$ gauge theories with N_f flavors

The generalization of our construction of the $SU(2)$ gauge theories to any value of N_c is straightforward. Instead of two parallel D5-branes, we have N_c parallel D5-branes, still between two other branes, which we can choose to have charges (p_1, q_1) and (p_2, q_2) for large values of x_3, and then for large negative values of x_3 they will have charges $(p_1, q_1 - N_c)$ and $(p_2, q_2 + N_c)$. The S_{N_c} Weyl symmetry corresponds, as usual, to permuting the D5-branes, and the $U(1)$ part of the naive gauge group is projected out as in the $SU(2)$ case (and as in [3]).
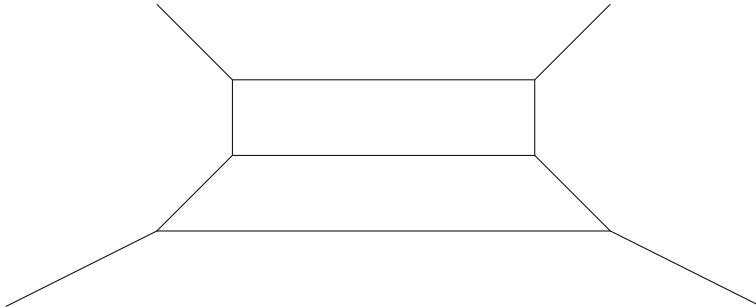

Figure 7: A point on the Coulomb branch of the pure $SU(3)$ gauge theory.

For example, a particular construction of the pure $SU(3)$ gauge theory is depicted in figure 7. The prepotential for this theory was computed in [10]. As for $SU(2)$, we can identify $1/g_0^2$ with the length of the D5-branes when they are all together. The 5D monopoles, whose tensions are given by linear combinations of derivatives of the prepotential, again correspond to D3-branes stretching over finite surfaces in the $x_3 - x_7$ plane. The area of these surfaces agrees with the known formulas for the monopole tensions (it is clear from the figures and from the generalizations of (3.3) that they are always piecewise quadratic in the x_3 positions of the D5-branes, which are related to the Cartan scalars ϕ_i). However, in these configurations we can no longer simply identify the W boson masses with the distances between the D5-branes. This is not too surprising since the other finite branes in these constructions should be just as important as the D5-branes. For instance, at the $SU(2)$ strong coupling fixed point of figure 5(c), there is a symmetry between the D and NS branes. The "polymers" describing the gauge theories which we find here are analogs of the Seiberg-Witten curve [26] of 4D $N = 2$ gauge theories, which was constructed in a similar way in [3].

As we did in the $SU(2)$ case, we can take the strong coupling limit of all these constructions, and obtain strong coupling fixed points for the pure $SU(N_c)$ gauge theories, of the general form discussed in §3.2. All these fixed points (corresponding to different asymptotic branes) are described by an intersection of four half 5-branes, so their global symmetry is of rank one. Generally, only a $U(1)$ global symmetry is visible, except when we choose two of the outgoing branes to be parallel, and then the visible global symmetry is $SU(2)$. Also, as in the $SU(2)$ case, there is at least one particular choice of asymptotic branes for which a Fayet-Iliopoulos-like deformation is visible at the strong coupling fixed point (corresponding to moving the branes apart in x_4, x_5 or x_6), given by $(p_i, q_i) = \{(-1, N_c - 1), (1, 1), (1, 1 - N_c), (-1, -1)\}$.

Next, we can add quarks (charged hypermultiplets) to these theories. As in previous brane constructions, there are two ways to do this – either by adding D7-branes in the 01245689 directions, which intersect the D5-branes giving rise to the gauge group (as in [1], but D7-branes affect the asymptotic space-time geometry so their analysis is more complicated), or by adding semi-infinite D5-branes parallel to the ones we have (as in [1,3]). We will choose the second construction since its analysis is simpler (though the Higgs branches are difficult to see in this choice, as discussed below). In this construction, each additional flavor corresponds to an additional semi-infinite D5-brane. As discussed in the previous section, each such additional brane adds one real parameter to the theory,
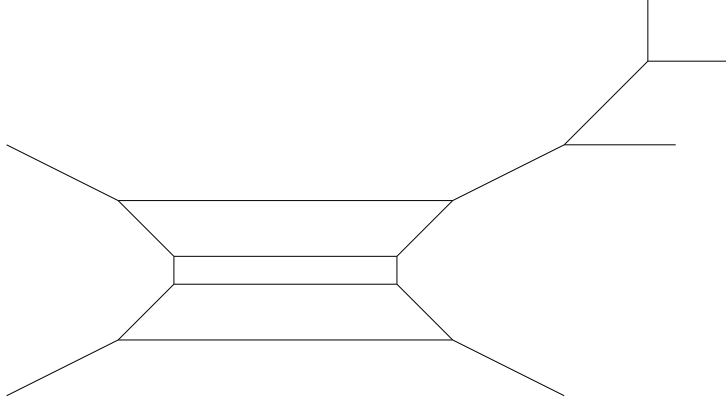
Figure 8: $SU(4)$ gauge theory with two (massive) flavors in five dimensions.

which it is natural to interpret as the (real) mass of the corresponding quark hypermultiplet (which is in the $\mathbf{N_c}$ representation).

An example of this type of constructions appears in figure 8. Note that for $N_c > 2$ and odd values of N_f, these theories have a $\mathbb{Z}_2$ anomaly [10] and cannot be properly defined as 5D gauge theories. The brane constructions of these theories, however, seem to be consistent, so the anomaly is probably cancelled by higher dimensional effects (as in [27,11]), but we have not checked this carefully.

As we did for the pure gauge theories, we can compute the monopole tensions by looking at areas in the brane construction, and we find agreement with the known field theory results.
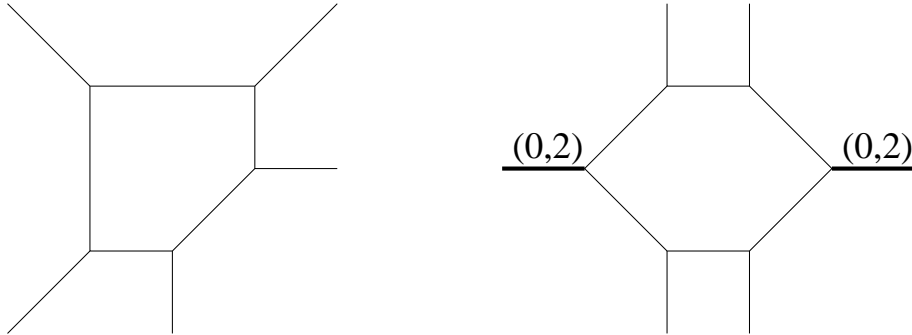

Figure 9: $SU(2)$ gauge theory with one massless quark at infinite coupling (and finite ϕ), and with four massless quarks at finite coupling (and finite ϕ).

The simplest examples are $SU(2)$ gauge theories, two of which (with massless quarks) are drawn in figure 9. In the first example, corresponding to $N_f = 1$ at the strong coupling fixed point, we find a tension of $\frac{7}{8}\phi^2$ (with the same normalization we used for the pure

gauge theory), as expected since the coefficient of the ϕ^2 term should be proportional to $8 - N_f$. In the second example, which is $N_f = 4$ at finite coupling, we find a tension of $1/g_0^2\phi + \frac{1}{2}\phi^2$, again as expected. Note that in this case only an $SU(2) \times SU(2)$ non-Abelian factor of the global symmetry is visible in the brane construction, even though we expect to have an $SO(8)$ global flavor symmetry at low energies.

As we did for the pure gauge theories, we can take the gauge coupling to infinity and find strong coupling fixed points at the origin of moduli space. With N_f flavors, these fixed points correspond to an intersection of $4 + N_f$ half 5-branes. As before, it is not clear if fixed points having the same Coulomb branch but different asymptotic 5-branes are the same or not. In a similar way, we can also construct $SU(N_c)$ theories by using N_c parallel (p, q) 5-branes, and again it is not clear if the fixed points related to these are the same as the previous ones or not. Brane configurations corresponding to SO and USp gauge theories (analyzed in [10]) may be constructed in a similar way by adding orientifolds (as in [17,18]) but will not be discussed here.
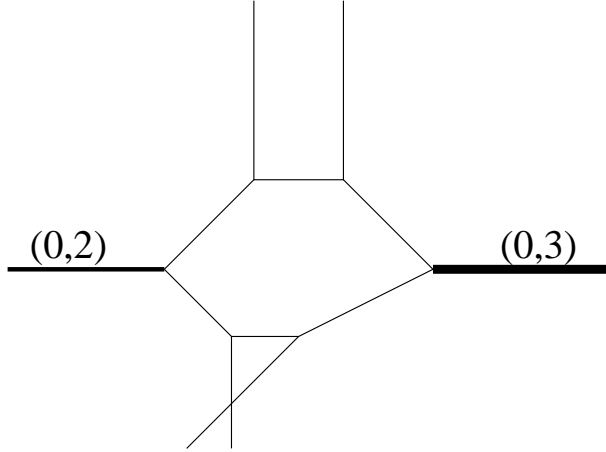

Figure 10: An attempt to construct the $SU(2)$ theory with $N_f = 5$.

Brane constructions of the type discussed above exist if (and only if) $N_f \le 2N_c$. If we try to add more flavors, we find that we must have more intersections of 5-branes than the ones we had before along the Coulomb branch, as in figure 10. It is then natural to assume that the field theory is no longer just the $SU(N_c)$ theory with N_f flavors, which does not have a positive definite metric for $N_f > 2N_c$ and $N_c \ge 3$ [10], but that there are additional degrees of freedom corresponding to the additional finite branes in the construction. For $N_c = 2$ consistent field theories are argued to exist also for $N_f = 5, 6, 7, 8$ [7], but we do not know how to see this in the brane construction (in which $N_c = 2$ does not appear

to be special). This may be related to the exceptional global symmetries of the strong coupling fixed points of these theories, though the global symmetries are not always visible in the brane construction. In any case, the strong coupling fixed points exist in the brane construction (as defined above) for any values of N_f and N_c, though for $N_f > 2N_c$ they can no longer be deformed into $SU(N_c)$ gauge theories. We conjecture that these fixed points correspond to new superconformal field theories.

So far we have not discussed the Higgs branches of the gauge theories we were considering, which should exist for $N_f \geq 2$. These branches are, in fact, difficult to see when the flavors come from semi-infinite 5-branes (and also in the analogous situation of [3]). Starting from the standard construction of quark flavors in [1], going to the constructions we describe involves moving branes away to infinity in the x_7 direction (x_6 in the notation of [1]), and the Higgs branches apparently go off to infinity as well. Note that we expect Higgs branches in our construction to be related to moving branes in the $x_{4,5,6}$ directions, since the $SU(2)_R$ symmetry should be broken along them. Since none of the finite 5-brane segments of our construction can move in these directions, it seems that the Higgs branch is, in fact, related to the semi-infinite 5-branes, which have low-energy 6 dimensional field theories. The relation between the five dimensional low-energy physics and the six dimensional physics is, therefore, more complicated here (and in [3]) than in [1], and we will not discuss it further here.

3.4. Reproducing the $SU(2)$ flows

In this section we present an application of the brane constructions to an analysis of the $SU(2)$ gauge theories with $N_f \leq 1$. The strong coupling fixed point of the $N_f = 1$ theory (called the E_2 theory) has two deformations, which may be interpreted as the quark mass and the gauge coupling. The analysis of these deformations was performed in [8,9], where it was found that one can flow to two different fixed points of the $N_f = 0$ theory, which were denoted E_1 and $\tilde{E}_1$, and that one can flow from the latter to an E_0 fixed point (which has a one dimensional Coulomb branch but no gauge theory interpretation).

All of these results may be rederived in the brane construction, as described in figure 11, which is the brane realization of figure 1 in [8]. The E_1 and $\tilde{E}_1$ theories are both special cases of the strong coupling fixed points of the pure $SU(2)$ gauge theory described in §3.1, with different asymptotic branes. The E_0 theory is realized as an intersection of three half 5-branes, so it has (using the results of §3.2) no parameters, as expected [8]. The three 5-branes may be chosen to have $(p_i, q_i) = \{(1,1), (-2,1), (1,-2)\}$.
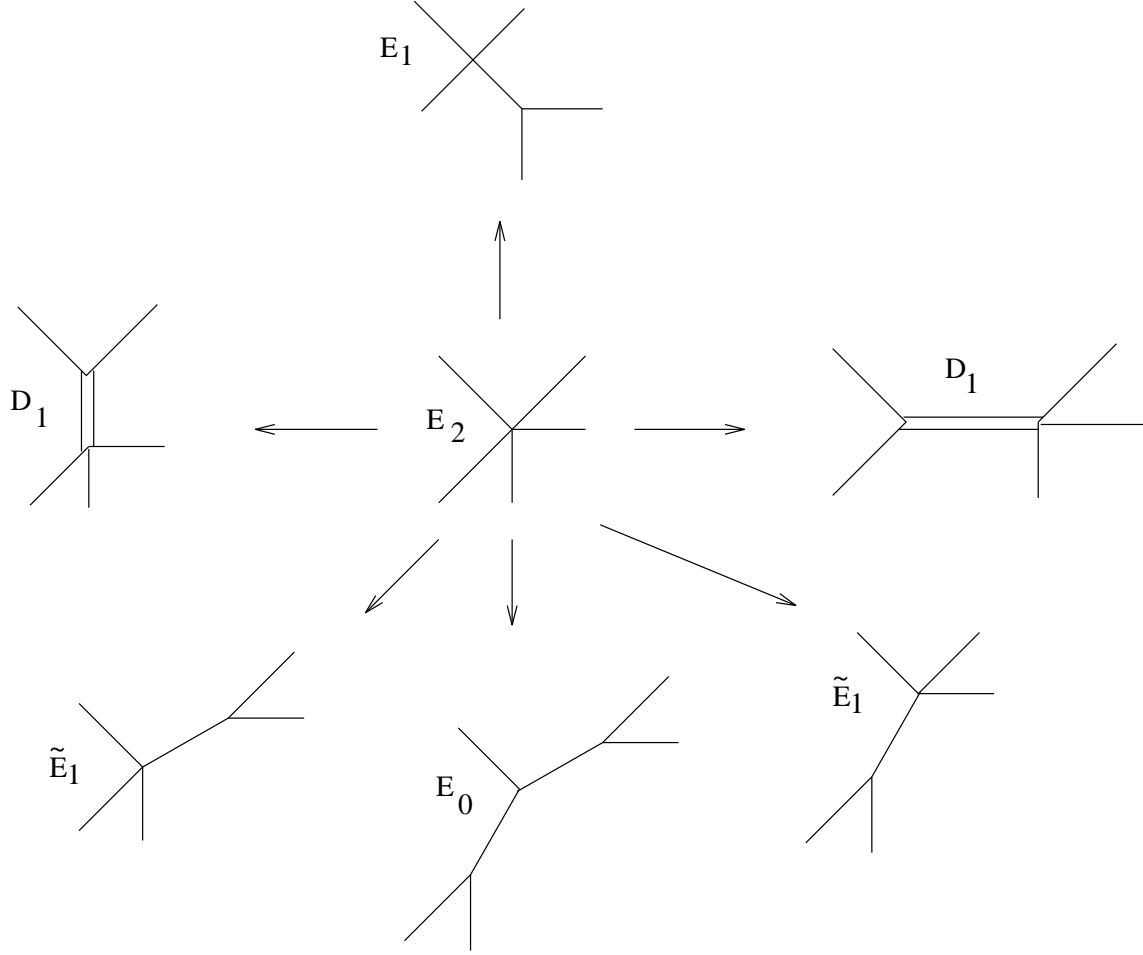
Figure 11: The E_2 fixed point and some flows in parameter space away from it.
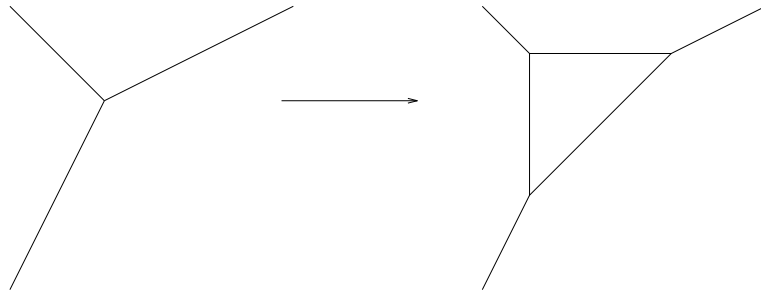

Figure 12: The Coulomb branch of the E_0 theory.

For most intersections of three half 5-branes there is no Coulomb branch, but in this case there is a Coulomb branch emanating from the E_0 point, as shown in figure 12. The other intersection points in figure 11 all have no Coulomb branches emanating from them, and appear to correspond to trivial theories.

Generic fixed points, as described in §3.2, will probably not correspond in any obvious way to gauge theories, like the E_0 theory. It is not clear if in general they can also be reached by flows from the $SU(N)$ fixed points. In any case, we are able to analyze their deformations and the flows between them, as described in the previous sections.

3.5. $SU(n) \times SU(m)$ gauge groups

Other field theory results we can easily reconstruct in the brane picture pertain to products of gauge groups. It was argued in [10] that products of simple gauge groups (with matter charged under both groups) cannot have strong coupling fixed points, since the effective coupling for one factor of the gauge group would become negative along the Coulomb branch corresponding to the other group.
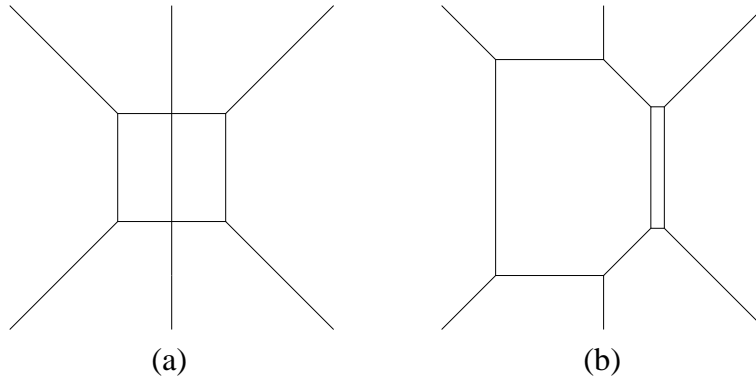

(a)   (b)

Figure 13: The $SU(2) \times SU(2)$ theory. Figure (a) shows a point on the Coulomb branch, while figure (b) shows what happens when we go along the Coulomb branch of one of the groups and not the other.

How do we see this in the brane constructions ? These are a generalization of the constructions analyzed in [3], and an example for $SU(2) \times SU(2)$ is described in figure 13. Naively, the theory in figure 13(a) is an $SU(2) \times SU(2)$ gauge theory with a hypermultiplet in the $(2, 2)$ representation, at a point on the Coulomb branch emanating from this theory's infinite coupling fixed point. However, we see in figure 13(b) that if we try to go far along the Coulomb branch of one of the $SU(2)$ groups and not of the other, the description of the other $SU(2)$ group breaks down. In fact, we seem to get its continuation beyond infinite coupling, as we discussed in §3.1. The brane configuration continues to describe some low-energy field theory, but we can no longer describe it simply as a $SU(2) \times SU(2)$ field theory.

In fact, the $SU(2) \times SU(2)$ theory described here appears very similar to the $SU(3)$ theory with $N_f = 2$ massless quarks, which is of the type described in §3.3. This is what we get if we turn the figure by 90 degrees, and replace NS branes with D branes[6]. Thus, the strong coupling fixed point that we naively attributed to the $SU(2) \times SU(2)$ theory is really the strong coupling fixed point of the $SU(3)$ theory with $N_f = 2$. The $SU(3)$ description makes sense on the whole Coulomb branch, while the $SU(2) \times SU(2)$ description does not. It should be interesting to understand the exact relationship between these two theories, and the generalization of this statement to arbitrary gauge groups.

4. From 5D Fixed Points to 3D Fixed Points

In this section we combine the methods of the previous two sections, and relate the 5D $N = 1$ superconformal theories constructed in §3 to 3D $N = 2$ superconformal theories. As in §2, this is done by stretching finite 3-branes in the 0126 directions between 5-branes, but now instead of using just NS 5-branes to bound the 3-branes, we will use "polymeric" 5-branes of the type considered in the previous section. Since the branes we use are a subset of the ones described in §2, this configuration preserves 1/8 of the supersymmetry, corresponding to $N = 2$ in three dimensions.

Consider any configuration of 5-branes like those described in the previous section, filling the 01289 directions and corresponding to a "polymer" in the $x_3 - x_7$ plane. A 3-brane in the 0126 directions can end on any (p, q) 5-brane, so, in particular, it can end at any point on the "polymer" and have flat directions corresponding to moving along the "polymer". In order to have a 3D theory we want the 3-brane to be finite in the x_6 direction, and if we want to preserve the flat directions corresponding to moving along the "polymer" we need to have two copies of the same "polymer" at two different values of x_6. Now, if we stretch a 3-brane (or several 3-branes) between these "polymers" we get a 3D $N = 2$ theory, with flat directions corresponding to moving along the "polymer".

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[6] Note that the configurations we discuss in this section are not invariant under general rotations in the $x_3 - x_7$ plane, since our choice of the unbroken supersymmetry is not consistent with such rotations, but rotations by 90 degrees are equivalent to the S generator of $SL(2, \mathbb{Z})$ as far as the unbroken supersymmetry is concerned, so they are expected not to change the strong coupling fixed points.

When the 3-brane is at a generic point on the "polymer" it ends (on both sides) on some (p,q) 5-brane, with p and q relatively prime[7]. This configuration is $SL(2,\mathbb{Z})$-dual to a 3-brane ending on both sides on a NS 5-brane, which is described (at low energies) by the $U(1)$ $N = 4$ gauge theory, as discussed in [1]. The massless fields are thus in a (p,q)-vector multiplet, which is related by electric-magnetic duality in the D3-brane to the standard vector multiplet (for a D3-brane between two D5-branes, this multiplet may be described as a hypermultiplet, as in [1], but its description in terms of a vector multiplet is more natural here). The motion along the (p,q) 5-brane (along the "polymer" and in the x_8 and x_9 directions) corresponds to the Coulomb branch of this theory. From the $N = 2$ point of view there is one (p,q) vector multiplet and one chiral multiplet, describing the x_{89} position of the 3-brane, which will exist (and be decoupled) in all configurations of this type. For N_c D3-branes together, all these fields are enhanced to adjoints of $U(N_c)$.

Things get more interesting when the 3-brane approaches some intersection of 5-branes. Along each branch separately, there is a massless (p,q) vector multiplet. The vector multiplets corresponding to 5-branes with different values of (p,q) are not mutually local (they are related by $SL(2,\mathbb{Z})$ transformations in the field theory of the 3-brane). At the intersection point we thus expect to get some non-trivial superconformal field theory, at least when we have more than one D3-brane so that there are charged particles (W bosons) under all the mutually non-local vector multiplets (as in similar four dimensional cases with $N = 2$ [28,29] and $N = 1$ [30] supersymmetry). Each non-trivial 5D SCFT described in the previous section thus gives rise to a 3D $N = 2$ SCFT as well. In fact, we get an infinite series of such SCFTs, one for every value of N_c.

Even trivial 5D constructions of the type described in §3 give rise to interesting 3D $N = 2$ SCFTs. For instance, the field theory corresponding to a D3-brane moving on the "polymer" described in figure 4 has three Coulomb-like branches, which are all non-local with respect to each other. Along each Coulomb branch the low-energy theory is an $N = 4$ theory, but with different $SU(2)_R \times SU(2)_L$ global symmetries (one of the $SU(2)$ factors corresponds to rotations in x_8, x_9 and the direction in the $x_3 - x_7$ plane along the "polymer," while the other corresponds to rotations in x_4, x_5 and the direction in the $x_3 - x_7$ plane orthogonal to the "polymer"). The breaking of $N = 4$ to $N = 2$ is felt at low energies only at the intersection point of the Coulomb branches.

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[7] A 3-brane ending on a 5-brane with p and q not relatively prime was conjectured in [2] to be described by a $\mathrm{tr}(X^k)$ superpotential for an adjoint field X, but we will not discuss this here.

A possible generalization of this construction still utilizes the same "polymers" on both sides, but constructed on one side from branes spanning the 01245 direction, and on the other side from branes spanning the 01289 direction (as well as the directions corresponding to the "polymer"). The supersymmetry in the 3D theory is then still $N = 2$, and the only difference from the previous case is that the scalar corresponding to the x_{89} position of the D3-brane is now massive, so along the Coulomb branches we have the pure $N = 2$ SYM theory.

Another possible generalization is to theories with two different "polymers" on both sides of the D3-branes, but with some of their "line segments" still overlapping (in the $x_3 - x_7$ plane). For instance, we could have the configuration of figure 4 on one side, and the same configuration shifted up in x_3 on the other side. In this case, the Coulomb branch corresponding to the D3-brane moving along the NS 5-brane still remains, but it ends when the NS 5-brane ends on one side, and there seem to be no other branches emanating from this end point. Presumably, such end points also correspond to interesting 3D $N = 2$ superconformal field theories.

Finally, we would like to discuss the implications of our discussion here for the conjecture of [6] regarding chiral symmetry in the brane construction. For the 3D $N = 2$ theories, this conjecture is that when N_f D5-branes overlap (in $4 + 1$ dimensions) with a NS 5-brane, there is actually an $SU(N_f) \times SU(N_f)$ gauge symmetry on the 5-branes, which is a chiral symmetry for the D3-branes ending on these D5-branes. Following our discussion of section 3, the intersection of the N_f D5-branes with the NS 5-brane would seem to be some superconformal theory (though the 5D part of this theory may be trivial, since there is no Coulomb branch coming out of the intersection point), and it is not obvious if it can really be described in terms of an $SU(N_f) \times SU(N_f)$ gauge symmetry.
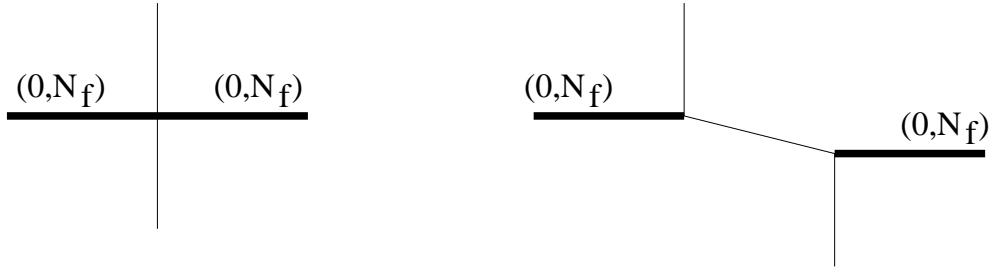

Figure 14: The 5-brane configuration conjectured to correspond to chiral symmetry, and a deformation of this configuration which splits the D5-branes.

However, unlike the 4D case discussed in [6], here we can split the N_f D5-branes along the NS 5-brane, as shown in figure 14. Then, it seems clear that there is indeed an $SU(N_f) \times SU(N_f)$ gauge symmetry in the 5+1 dimensional gauge theory of the D5-branes. However, it is no longer clear what field theory corresponds to a D3-brane ending on this configuration. As discussed above, the theory of a D3-brane ending at the intersection point between the NS 5-brane and the N_f D5-branes corresponds to some superconformal fixed point, which does not seem to correspond to a gauge theory with N_f massless quarks (and no anti-quarks), as suggested by [6] (though it might be some deformation of this theory). The issue of chiral symmetry in the brane configurations deserves further investigation.

Note Added

As this paper was being completed, we received [31], which has some overlap with our discussion in section 2.

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